Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4+6x}{2}$ (d) $\dfrac{4\times 6x}{2}$ (e) $\dfrac{3x+xy}{x}$

(f) $\dfrac{4x+9y}{2x+3y}$ (g) $\dfrac{4x+6y}{6x+9y}$ (h) $\dfrac{5xy+6y^2}{10x+12y}$

(i) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ (j) $\dfrac{x-3}{3-x}$ (k) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c)* $\frac{1}{2}k(k+1) + (k+1)$ **(d)*** $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$ (b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

Further Maths Only

(e)* $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2}$ **(f)*** $\dfrac{\dfrac{x}{2\sqrt{1-x}} + \sqrt{1-x}}{x^2}$

Exercise 1.4

(1) (a) $5x+3y+7x-3y = \underset{\sim}{12x}$

(b) $3x^2+4xy+y^2+x^2-4xy-y^2$

$= \underset{\sim}{4x^2}$

(c) $\dfrac{4+6x}{2} = \underset{\sim}{2+3x}$

(d) $\dfrac{4\times 6x}{2} = \dfrac{24x}{2} = \underset{\sim}{12x}$

(e) $\dfrac{3x+xy}{x} = \dfrac{x(3+y)}{x} = \underset{\sim}{3+y}$

(f) $\dfrac{4x+9y}{2x+3y}$ cannot be simplified

(g) $\dfrac{4x+6y}{6x+9y} = \dfrac{2(2x+3y)}{3(2x+3y)} = \underset{\sim}{\dfrac{2}{3}}$

(h) $\dfrac{5xy+6y^2}{10x+12y} = \dfrac{y(5x+6y)}{2(5x+6y)} = \underset{\sim}{\dfrac{y}{2}}$ (or $\frac{1}{2}y$)

(i) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ cannot be simplified

(j) $\dfrac{x-3}{3-x} = \dfrac{x-3}{-(x-3)} = \underset{\sim}{-1}$

(k) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2} = \dfrac{x^2-2xy-y^2}{-(x^2-2xy-y^2)} = \underset{\sim}{-1}$

(2) (a) $\frac{ax}{b} = \frac{py}{qz}$

$(\times b, \div a) \quad x = \frac{bpy}{aqz}$

(b) $\frac{3\pi ax}{b} = \frac{4y^2}{qz}$

$(\times b, \div 3\pi a) \quad x = \frac{4by^2}{3\pi aqz}$

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(3) (a) $\frac{2\pi x}{ab} \div \frac{1}{3}\pi r^3 = \frac{2\pi x}{ab} \div \frac{\pi r^3}{3}$

$= \frac{2\pi x}{ab} \times \frac{3}{\pi r^3}$

$= \frac{6\pi x}{\pi ab r^3}$

$= \frac{6x}{abr^3}$

(b) $\frac{2\pi h^2}{rb} \div \frac{4\pi hr^2}{3} = \frac{2\pi h^2}{rb} \times \frac{3}{4\pi hr^2}$

$= \frac{6\pi h^2}{4\pi bhr^3}$

$= \frac{3h}{2br^3}$

(4) (a) $(x-3)^2 + 5(x-3)^3 = (x-3)^2\left[1 + 5(x-3)\right]$

$= (x-3)^2(1 + 5x - 15)$

$= \underline{(5x-14)(x-3)^2}$

(b) $4x(2x+1)^3 + 5(2x+1)^4 = (2x+1)^3\left[4x + 5(2x+1)\right]$

$= (2x+1)^3(4x + 10x + 5)$

$= \underline{(14x+5)(2x+1)^3}$

(c) $\frac{1}{2}k(k+1) + (k+1) = \frac{1}{2}k(k+1) + \frac{2(k+1)}{2}$

$= \frac{1}{2}(k+1)\left[k+2\right]$

$= \underline{\frac{1}{2}(k+1)(k+2)}$

(d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2 = \frac{1}{6}k(k+1)(2k+1) + \frac{6}{6}(k+1)^2$

$= \frac{1}{6}(k+1)\left[k(2k+1) + 6(k+1)\right]$

$= \frac{1}{6}(k+1)(2k^2 + k + 6k + 6)$

$= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$

$= \frac{1}{6}(k+1)(2k+3)(k+2)$

$= \underline{\frac{1}{6}(k+1)(k+2)(2k+3)}$

$2k^2 + 7k + 6 = 2k^2 + 4k + 3k + 6$

$= 2k(k+2) + 3(k+2)$

$= (k+2)(2k+3)$

P: $2 \times 6 = +12$

A: $+7$

($+4$ and $+3$)

(5) (a) $\dfrac{x^2+6x+8}{x^2-x-6} = \dfrac{(x+2)(x+4)}{(x+2)(x-3)}$

$= \dfrac{x+4}{x-3}$

(b) $\dfrac{3x^2-2x-8}{x^2-4} = \dfrac{(x-2)(3x+4)}{(x+2)(x-2)} = \dfrac{3x+4}{x+2}$

$3x^2-2x-8 = 3x^2-6x+4x-8$

$= 3x(x-2)+4(x-2)$

$= (x-2)(3x+4)$

P: $3x \times -8 = -24$

A: -2

-6 and $+4$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3} = \dfrac{(x+3)[(x+3)-2]}{(x+3)(x-1)}$

$= \dfrac{(x+3)(x+1)}{(x+3)(x-1)}$

$= \dfrac{x+1}{x-1}$

(d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2} = \dfrac{x(2x-1)[(2x-1)-x]}{(x-1)^2}$

$= \dfrac{x(2x-1)(2x-1-x)}{(x-1)^2}$

$= \dfrac{x(2x-1)(x-1)}{(x-1)^2}$

$= \dfrac{x(2x-1)}{x-1}$

(4)(e) $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2} = \dfrac{\dfrac{x^2 - \left(\sqrt{x^2+1}\right)^2}{\sqrt{x^2+1}}}{x^2}$

$= \dfrac{\left[x^2 - \left(x^2+1\right)\right]}{\sqrt{x^2+1}} \div \dfrac{x^2}{1}$

$= \dfrac{\left(x^2 - x^2 - 1\right)}{\sqrt{x^2+1}} \times \dfrac{1}{x^2}$

$= \dfrac{-1}{x^2\sqrt{x^2+1}}$

(f) $\dfrac{\dfrac{x}{2\sqrt{1-x}} + \sqrt{1-x}}{x^2} = \dfrac{\dfrac{x + 2\left(\sqrt{1-x}\right)^2}{2\sqrt{1-x}}}{x^2}$

$= \dfrac{\left[x + 2(1-x)\right]}{2\sqrt{1-x}} \div \dfrac{x^2}{1}$

$= \dfrac{(x + 2 - 2x)}{2\sqrt{1-x}} \times \dfrac{1}{x^2}$

$= \dfrac{2-x}{2\sqrt{1-x}} \times \dfrac{1}{x^2}$

$= \dfrac{2-x}{2x^2\sqrt{1-x}}$