Exercise 1.6

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Further Maths Only

7* Simplify $\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$

Exercise 1.6

① (a) $\frac{1}{x} = x^{-1}$ (b) $\frac{1}{x^5} = x^{-5}$ (c) $\sqrt[5]{x} = x^{1/5}$

(d) $\sqrt[3]{x^5} = (\sqrt[3]{x})^5 = x^{5/3}$

(e) $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

(f) $\frac{1}{\sqrt{x^3}} = \frac{1}{x^{3/2}} = x^{-3/2}$

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② (a) $x^{-4} = \frac{1}{x^4}$ (b) $x^0 = 1$ (c) $x^{1/6} = \sqrt[6]{x}$

(d) $x^{3/4} = \sqrt[4]{x^3}$ $\left[\text{or } (\sqrt[4]{x})^3\right]$

(e) $x^{-3/2} = \frac{1}{x^{3/2}} = \frac{1}{\sqrt{x^3}}$ $\left[\text{or } \frac{1}{(\sqrt{x})^3}\right]$

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③ (a) $4\sqrt[3]{x} = 4x^{1/3}$

(b) $\frac{3}{x^2} = 3x^{-2}$

(c) $\frac{5}{\sqrt{x}} = \frac{5}{x^{1/2}} = 5x^{-1/2}$

(d) $\frac{1}{2x^3} = \frac{1}{2}x^{-3}$

(e) $6 = 6x^0$

(4) (a) $x^3\left(x+\frac{1}{x}\right) = \underline{x^4 + x^2}$

(b) $\frac{x^4+1}{x^2} = \frac{x^4}{x^2} + \frac{1}{x^2} = \underline{x^2 + x^{-2}}$

(c) $x^{-5}\left(x+\frac{1}{x^2}\right) = x^{-5}\left(x+x^{-2}\right) = \underline{x^{-4} + x^{-7}}$

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(5) (a) $\sqrt{75} = \sqrt{25\times 3} = \sqrt{25}\sqrt{3} = \underline{5\sqrt{3}}$

(b) $\sqrt{180} = \sqrt{36\times 5} = \sqrt{36}\sqrt{5} = \underline{6\sqrt{5}}$

(c) $\frac{12}{\sqrt{6}} = \frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{12\sqrt{6}}{6}$

$= \underline{2\sqrt{6}}$

(d) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \underline{\frac{\sqrt{5}}{5}}$

(e) $\frac{3}{\sqrt{12}} = \frac{3}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$

$= \frac{3\sqrt{12}}{12}$

$= \frac{1}{4}\sqrt{12}$

$= \frac{1}{4}\sqrt{4\times 3} = \frac{1}{4}\sqrt{4}\sqrt{3} = \frac{2}{4}\sqrt{3} = \underline{\frac{1}{2}\sqrt{3}}$

$\left(\text{or } \frac{\sqrt{3}}{2}\right)$

(6) (a) $\frac{1}{\sqrt{2}-1} = \frac{1}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

difference of two squares

$= \frac{\sqrt{2}+1}{2-1}$

$= \underline{\sqrt{2}+1}$

$(\text{or} \quad 1+\sqrt{2})$

(b) $\frac{2}{\sqrt{6}-2} = \frac{2}{(\sqrt{6}-2)} \times \frac{(\sqrt{6}+2)}{(\sqrt{6}+2)}$

$= \frac{2(\sqrt{6}+2)}{6-4}$

$= \frac{2(\sqrt{6}+2)}{2}$

$= \underline{\sqrt{6}+2}$

$(\text{or} \quad 2+\sqrt{6})$

(c) $\frac{6}{\sqrt{7}+2} = \frac{6}{(\sqrt{7}+2)} \times \frac{(\sqrt{7}-2)}{(\sqrt{7}-2)}$

$= \frac{6(\sqrt{7}-2)}{7-4}$

$= \frac{6(\sqrt{7}-2)}{3}$

$= 2(\sqrt{7}-2)$

$= \underline{2\sqrt{7}-4}$

$(\text{or} \quad -4+2\sqrt{7})$

(d) $\frac{1}{3+\sqrt{5}} = \frac{1}{(3+\sqrt{5})} \times \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$

$= \frac{3-\sqrt{5}}{9-5}$

$= \frac{3-\sqrt{5}}{4}$ $\left[\text{or} \quad \frac{3}{4} - \frac{1}{4}\sqrt{5}\right]$

(e) $\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{(\sqrt{6}-\sqrt{5})} \times \frac{(\sqrt{6}+\sqrt{5})}{(\sqrt{6}+\sqrt{5})}$

$= \frac{\sqrt{6}+\sqrt{5}}{6-5}$

$= \frac{\sqrt{6}+\sqrt{5}}{1}$

$= \sqrt{6}+\sqrt{5}$ $\left[\text{or} \quad \sqrt{5}+\sqrt{6}\right]$

FURTHER MATHEMATICS ONLY

(7) $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{100}+\sqrt{99}}$

$\frac{1}{\sqrt{2}+\sqrt{1}} = \frac{1}{(\sqrt{2}+\sqrt{1})} \times \frac{(\sqrt{2}-\sqrt{1})}{(\sqrt{2}-\sqrt{1})}$

$= \frac{\sqrt{2}-\sqrt{1}}{2-1}$

$= \frac{\sqrt{2}-\sqrt{1}}{1}$

$= \sqrt{2}-\sqrt{1}$

Similarly $\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$$= \frac{\sqrt{3}-\sqrt{2}}{3-2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{1}$$

$$= \sqrt{3}-\sqrt{2}$$

So $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{100}+\sqrt{99}}$

$= (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \cdots + (\sqrt{99}-\sqrt{98}) + (\sqrt{100}-\sqrt{99})$

$= -\sqrt{1} + \sqrt{100}$

$= -1 + 10$

$= \underline{9}$